- Unit — 2 -

- Matrix And Determinants -

⇒ Define Matrix.

Ans. A matrix is defined as rectangular array of elements arranged in rows and collumns.

The following example illustrates how much matrixes are useful in storing for the six numbers, 2, 0, 7, 3, 5, 1 in three rows and two columns.

$$\begin{array}{cc|l} & & \\ \end{array}\begin{bmatrix} 2 & 0 \\ 7 & 3 \\ 5 & 1 \end{bmatrix} \begin{array}{l} \leftarrow \text{First Row} \\ \leftarrow \text{Second Row} \\ \leftarrow \text{Third Row} \end{array}$$

↑ First Column ↑ Second Column.

Generally, each number in a matrix is called an element, denoted by small letters, a, b, c ... and single letters A, B, C ..... may be used to name matrixes.

Order of the Matrix — If a matrix has three rows and three columns, then we call it a 3×3 matrix and the order of the matrix is 3×3. When a matrix has m rows and n columns, then the order of the matrix is m × n. for example—

i) $A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$

(Rows × Columns)

∴ The order of the A matrix is m × n.

ii) $B = \begin{bmatrix} 5 & 6 \\ 0 & -3 \end{bmatrix}$

(Rows × Columns)

2×2

∴ The order of the B matrix is 2×2.

iii) $C = \begin{bmatrix} 2 & 1 & 0 & 2 \\ 3 & 0 & 5 & 1 \end{bmatrix}$

(Rows × Columns)

2×4

The order of the C matrix is 2×4.

iv) $D = \begin{bmatrix} 2 & -7 & 8 & 9 \end{bmatrix}$

(Rows × Columns)

1 × 4.

∴ The order of the D matrix is 1×4.

Importance of Matrix:—

1. A matrix is not just an aggregate of numbers, because in a given matrix each element has its assigned position in a particular row and column.

It is to be remembered that –

matrix → $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 2 & 1 \end{bmatrix}$ is not same as $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 2 \\ 3 & 1 & 1 \end{bmatrix}$.

2. Matrix A = Matrix B if they have the same order and each element of A is equal to the corresponding element of B.

3. If the matrix consist of only one column as $\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$, it is a column matrix or column vector.

4. If a matrix consists of only one ~~column~~ rows as $[a_1 \; b_1 \; c_1]$ it is a row matrix or row vector.

5. A matrix is said to be a zero or null matrix if an only if each of its element is zero.

Various types of Matrix:—

1. **Square Matrix:—** A matrix whose number of rows and number of columns are equal is called a square matrix. A and B Matrices are square matrix, but C is not a square matrix.

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}_{2\times2} ; \quad B = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}_{3\times3} ; \quad C = \begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}_{3\times2}$$

2. **Identity or Unit Matrix:—** The square matrix whose principal diagonal elements are 1 and all other elements are zero, is called an identity matrix or unit matrix. It is also noted that identity must always be a square matrix.

$$\therefore \; I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times2} ; \quad I_2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times3} \quad I_n = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & & & \\ 0 & 0 & \cdots & 1 \end{bmatrix}_{n\times n}$$

3. <u>Scalar Matrix</u> :- A matrix which has a common scalar element in the principal diagonal and zeros every-where else is known as scalar matrix.

A square matrix whose all elements except those in the main diagonal are zero and the diagonal elements are all equal is called a scalar matrix. For example—

i) $A = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}_{2\times 2}$ ii) $B = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}_{3\times 3}$.

4. <u>Diagonal Matrix</u> :- If all off-diagonal elements of a matrix are zero, the matrix is called diagonal matrix. For example—

$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$; $B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$

Both are diagonal matrix.

5. <u>Null Matrix</u> :- A matrix of any order whose all elements are zero, is called a null matrix and is denoted by 'O'. For example—

i) $A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}_{2\times 2}$ ii) $B = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}_{1\times 4}$.

6. <u>Symmetric Matrix</u> :- A matrix whose elements are such that if we interchange the corresponding rows and columns the matrix remains the same is called a symmetric matrix. In symmetric matrix therefore the elements of corresponding rows and columns are the same. For Example

$A = \begin{bmatrix} 3 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 4 & 2 \end{bmatrix}$ or $A = \begin{bmatrix} 4 & 0 & 2 \\ 0 & 1 & 5 \\ 2 & 5 & 3 \end{bmatrix}$.

— Operations On Matrixes —

1. Addition of Matrix :- Two matrices (Say A and B) are said to be conformable for addition, if and only if they have the same number of rows and columns, then the sum of A and B, written as A + B.

For example — 1. If $A = \begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$

then find $A + B = ?$

Solution— Matrix A & B are conformable for addition because both matrices have same order (2×2).

$$\therefore A + B = \begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix} + \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 1+5 & 2+6 \\ 3+7 & -4+8 \end{bmatrix} = \begin{bmatrix} 6 & 8 \\ 10 & 4 \end{bmatrix} \text{ Ans.}$$

2. $\begin{bmatrix} 10 & 13 & 1 & 7 \\ 2 & 3 & 1 & -1 \\ 2 & -4 & 0 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 3 & -1 & 1 & 0 \\ 1 & -2 & 0 & 1 \end{bmatrix}$ find addition.

$$= \begin{bmatrix} 10 & 13 & 1 & 7 \\ 5 & 2 & 2 & -1 \\ 3 & -6 & 0 & 3 \end{bmatrix} \text{ Ans.}$$

2. Subtraction of Matrix :- A and B matrices are said to be conformable for subtraction, if and only if they have the same number of rows and columns, then the subtraction of A and B, written as A − B.

For example → 1. If $A = \begin{bmatrix} 2 & 3 & -2 \\ 1 & 6 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 7 & -2 \\ 3 & 5 & 4 \end{bmatrix}$

then $A - B = ?$

Solution— Both matrices have same order. i.e, 2×3. So they are conformable for subtraction.

$$A - B = \begin{bmatrix} 2 & 3 & -2 \\ 1 & 6 & 8 \end{bmatrix} - \begin{bmatrix} -1 & 7 & -2 \\ 3 & 5 & 4 \end{bmatrix} = \begin{bmatrix} 3 & -4 & 0 \\ -2 & 1 & 4 \end{bmatrix} \text{ Ans.}$$

2. If $A = \begin{bmatrix} 2 & 0 \\ 3 & 1 \\ 5 & 6 \end{bmatrix}_{3\times 2}$ and $B = \begin{bmatrix} 7 & 8 \\ 5 & 3 \end{bmatrix}_{2\times 2}$ then $B - A = ?$

Solution – The matrices B and A are not conformable for substraction, because they have not same order. (i,e, – the order B matrix is 2×2 and the order of A-matrix is 3×2) –

3. If $A = \begin{bmatrix} 7 & 3 \\ 2 & 9 \end{bmatrix}$ $B = \begin{bmatrix} 4 & 2 \\ 3 & 5 \end{bmatrix}$ then $A - B = ?$

$$A - B = \begin{bmatrix} 3 & 1 \\ -1 & 4 \end{bmatrix}$$ Ans

4. If $A = \begin{bmatrix} 1 & 2 & 0 & 4 \\ 2 & 4 & -1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 & 0 & 3 \\ 1 & -1 & 2 & 3 \end{bmatrix}$, find – a matrix X of order 2×4. such that –

$$A - X = 3B.$$
$$\therefore -X = 3B - A$$
$$\therefore X = A - 3B.$$

$$= \begin{bmatrix} 1 & 2 & 0 & 4 \\ 2 & 4 & -1 & 3 \end{bmatrix} - 3\begin{bmatrix} 2 & 1 & 0 & 3 \\ 1 & -1 & 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 0 & 4 \\ 2 & 4 & -1 & 3 \end{bmatrix} - \begin{bmatrix} 6 & 3 & 0 & 9 \\ 3 & -3 & 6 & 9 \end{bmatrix} = \begin{bmatrix} -5 & -1 & 0 & -5 \\ -1 & 7 & 7 & -6 \end{bmatrix}$$ Ans

3. **Multiplication of Matrix :–** When the number of columns of A matrix is the same as the number of rows and another B matrix, then A is said to be conformable to B for the product AB and thereby the product AB is defined and denoted by AB. For example –

1. If $A = \begin{bmatrix} 2 & 5 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 6 \\ 3 & 2 \end{bmatrix}$ then find AB.

$$AB = \begin{bmatrix} 2 & 5 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 6 \\ 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \times 1 + 5 \times 3 & 2 \times 6 + 5 \times 2 \\ 3 \times 1 + 4 \times 3 & 3 \times 6 + 4 \times 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 + 15 & 12 + 10 \\ 3 + 12 & 18 + 8 \end{bmatrix} = \begin{bmatrix} 17 & 22 \\ 15 & 26 \end{bmatrix}$$ Ans.

2. If $A = \begin{bmatrix} 3 & 4 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 6 & 7 & 8 \\ 2 & 3 & 4 \\ 1 & 2 & 3 \end{bmatrix}$ find AB.

$$= \begin{bmatrix} 3 \times 6 + 4 \times 2 + 2 \times 1 \ , & 3 \times 7 + 4 \times 3 + 2 \times 2 \ , & 3 \times 8 + 4 \times 4 + 2 \times 3 \end{bmatrix}$$

$$= \begin{bmatrix} 18 + 8 + 2 \ , & 21 + 12 + 4 \ , & 24 + 16 + 6 \end{bmatrix}$$

$$= \begin{bmatrix} 28 & 37 & 46 \end{bmatrix}$$ Ans.

3. If $A = \begin{bmatrix} 3 & 4 \\ -2 & -3 \end{bmatrix}$ show that A^2 are identity-matrix.

$$A \times A = \begin{bmatrix} 3 & 4 \\ -2 & -3 \end{bmatrix} \times \begin{bmatrix} 3 & 4 \\ -2 & -3 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 3 \times 3 + 4 \times (-2) & 3 \times 4 + 4 \times (-3) \\ (-2) \times 3 + (-3) \times (-2) & (-2) \times 4 + (-3) \times (-3) \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 9 + (-8) & 12 + (-12) \\ (-6) + 6 & -8 + 9 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$ = Identity. Proved.